$v(t) = 80\cos(10t + 20^\circ)$ V
$i(t) = 15\cos(10t + 60^\circ)$ A

Find the instantaneous and average Power

$P_{INST} = v(t)\, i(t)$

$= \frac{V_m I_m}{2} \left\{ \cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i) \right\}$

$= \frac{80(15)}{2} \left(\cos(-40^\circ) + \cos(2\omega t + 80^\circ) \right)$

$$\boxed{P_{INST} = 459.6 + 600\cos(20t + 80^\circ) \text{ W}}$$

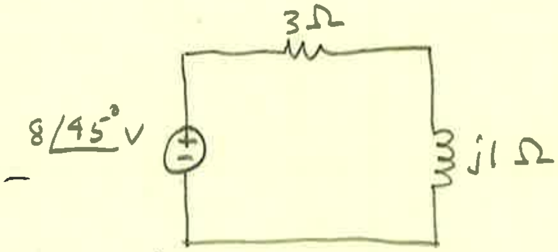
$P_{AVG} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$

$= \frac{80(15)}{2} \cdot \cos(-40^\circ)$

$$\boxed{P_{AVG} = 459.6 \text{ W}}$$

Chap 10 Lecture | AVERAGE POWER

calculate the average Power absorbed by the resistor and inductor + the power supplied by the source



$P_{AVG} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$i = \dfrac{8\angle 45^\circ}{3+j1} = \dfrac{8\angle 45}{3.162\angle 18.4^\circ} = 2.53\angle 26.6^\circ$ A

$V_R = iR = 7.59\angle 26.6^\circ$

$V_L = iZ_L = 2.53\angle 116.6^\circ$

$P_{Avg_R} = \frac{1}{2}(7.59)(2.53)\cos(26.6^\circ - 26.6^\circ) = \underline{\underline{9.6\,W}}$

$P_{Avg_L} = \frac{1}{2}(2.53)(2.53)\cos(116.6^\circ - 26.6^\circ) = \underline{\underline{0\,W}}$

$P_{source} = \frac{1}{2}(8)(2.53)\cos(45 - 26.6^\circ) = \underline{\underline{9.6\,W}}$

Example 10.1 | Nilsson 11^{th}

1) Calculate the average and reactive powers if:

$V = 100\cos(\omega t + 15^\circ)$ V
$I = 4\sin(\omega t - 15^\circ)$ A

Using the trig identity $\sin \omega t = \cos(\omega t - 90^\circ)$

$I = 4\cos(\omega t - 105^\circ)$ A

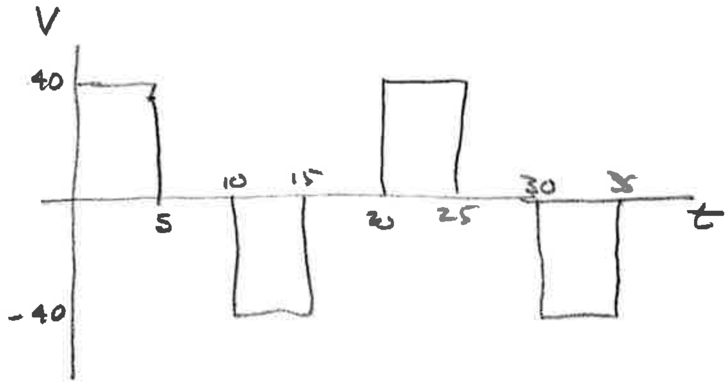
$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(100)(4)\cos(15-(-105)) = \boxed{-100\text{ W}}$

$Q = \frac{1}{2}V_m I_m \sin(\theta_v - \theta_i) = \frac{1}{2}(100)(4)\sin(15+105) = \boxed{173.21\text{ VARS}}$

2) Is the circuit delivering or absorbing average + reactive power?

Since average Power is negative - Delivering.
" Reactive " " positive - absorbing

a) Find the RMS value of the periodic waveform.



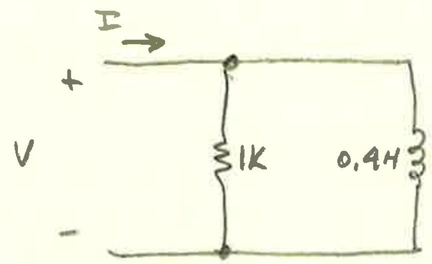
$$V_{RMS} = \sqrt{\frac{1}{T}\int^{T} V^2 dt}$$

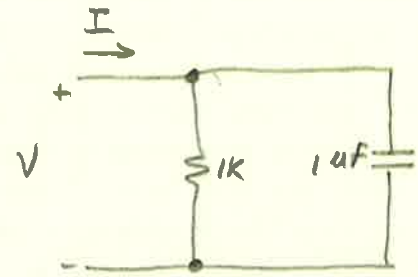
$$= \left[\frac{1}{20}\left(\int_0^5 40^2 dt + \int_{10}^{15} (-40)^2 dt\right)\right]^{1/2}$$

$$= \left[\frac{1}{20}(1600(5) + 1600(15-10)\right]^{1/2}$$

$$\boxed{= 28.28\ V_{RMS}}$$

b) $P_{Avg} = \frac{V^2}{R} = \boxed{20W}$





For $V = 10 \cos 2500t$, Find the power Factor for both circuit.

$I = \frac{V}{Z} = \frac{V}{1K \parallel j\omega L}$

$= \frac{10\angle 0^\circ}{707.11\angle 45^\circ}$

$= 14.14\angle -45^\circ$ mA

PF angle $= \theta_V - \theta_i$

$= 0 - (-45)$

$= 45^\circ$

$\boxed{PF = \cos 45^\circ = 0.707 \text{ Lagging}}$

$I = \frac{V}{Z} = \frac{V}{1K \parallel \frac{1}{j\omega C}}$

$= \frac{10\angle 0^\circ}{371.39\angle -68.20^\circ}$

$= 26.93\angle 68.20^\circ$ mA

PF angle $= \theta_V - \theta_i$

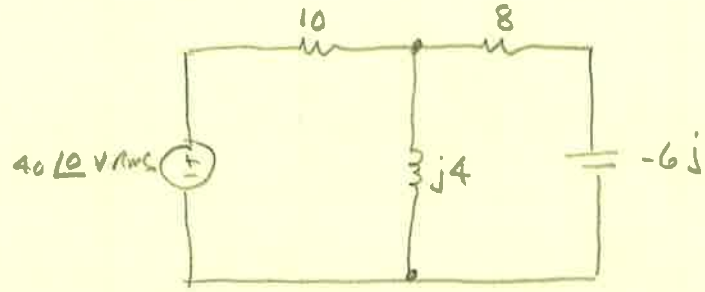
$= 0 - 68.20^\circ$

$= -68.20^\circ$

$\boxed{PF = \cos(-68.20) = 0.371 \text{ leading}}$

Note: $PF = \cos(\angle Z_{eq})$

Find PF as seen by the source



$Z = 10 + j4 \| (8-6j)$

$= 10 + \frac{j4(8-6j)}{j4+8-6j} = 10 + \frac{32j+24}{8-2j} = 10 + \frac{40\angle 53.1301}{8.2462\angle -14.0362^\circ}$

$Z = 10 + 4.8507\angle 67.1663^\circ = 10 + 1.8824 + 4.4706j$

$= 11.8824 + 4.4706j = 12.6955\angle 20.6182^\circ$

$PF = \angle Z = \cos(20.6182^\circ) = \boxed{.936 \text{ (lagging) } \angle \text{ of } Z \text{ is positive}}$

$P_{AVG\ source} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$ $\quad i = \frac{V}{Z} = 3.1507\angle -20.6182$

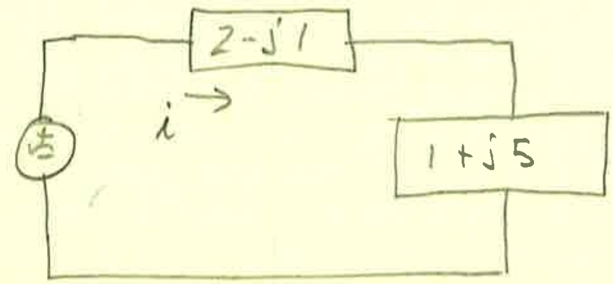
$= 40(3.1507)\cos(0-(-20.6182))$

$\boxed{= 118W}$

P. 440 HAYT 8^{th}

1) Find P_{avg} for Both Z's
2) " P_{APP} supplied by source
3) PF

$60\angle 0^\circ_{RMS}$



1) $i = \frac{V}{Z} = \frac{60\angle 0}{2-j1+1+j5} = \frac{60\angle 0}{3+j4} = 12\angle -53.1301^\circ\ A_{RMS}$

$V_{2-j1} = IZ = (12\angle -53.1301^\circ)(2-j1) = 26.8328\angle -79.6952^\circ\ V_{RMS}$

$V_{1+j5} = IZ = 61.1882\angle 25.56^\circ\ V_{RMS}$

$P_{Avg(2-j1)} = V_{RMS} I_{RMS} \cos(\theta_v - \theta_i) = \boxed{288W}$

$P_{Avg(1+j5)} = \boxed{144W}$

also, $P_{Avg_2} = I^2 |Z_{Real}| = (12)^2(2) = \underline{\underline{288}}$

$P_{Avg_1} = I^2 |Z_{real}| = (12)^2(1) = \underline{\underline{144}}$

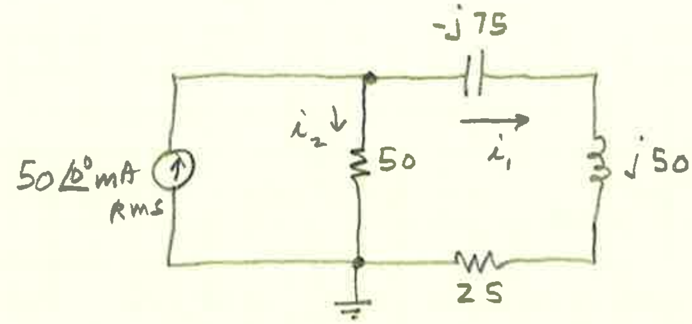
2) $P_{APP} = V_{RMS} I_{RMS} = 60(12) = \underline{\underline{720\ VA}}$

3) $PF = \cos(\theta_v - \theta_i) = \cos(0-(-53.1301^\circ)) = \boxed{0.6 \text{ lagging}}$

also, PF = $\angle$ of $Z = \frac{V}{I}$

$\angle Z = (2-j1)+(1+j5) = 3+j4 = 5\angle 53.1301^\circ$

also, $PF = \frac{P_{Avg}}{P_{APP}} = \frac{288+144}{720} = \underline{\underline{0.6}}$

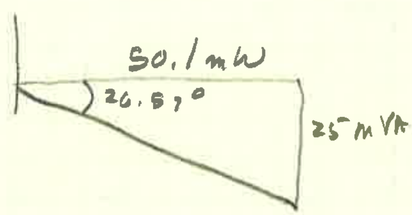


a) Find P_{AVG} + P_{REAC} for the source

$Z_{eq} = 50 // (-j75 + j50 + 25)$
$= \frac{50(25-25j)}{50+25-25j} = \frac{50(35.36\angle -45^\circ)}{79.06\angle -18.43^\circ}$
$= 22.36\angle -26.57^\circ\ \Omega$
$= 20 - j10\ \Omega$

$V_{source} = I_s Z_{eq} = 50\angle 0\ (22.36\angle -26.57^\circ)$
$= 1.12\angle -26.52^\circ$
$= 1 - .5j$

$P_{AVG} = V_{RMS} I_{RMS} \cos(\theta_V - \theta_i)$
$= 1.12(50mA)\cos(-26.52 - 0)$
$\boxed{P_{AVG} = -50.0\ mW}$



PF angle $= \angle Z = -26.57^\circ$
$P_{REAC} = V_{RMS} I_{RMS} \sin(\theta_V - \theta_i)$
$\boxed{P_{REAC} = +25\ mVAR}$

b) current Source is delivering $\boxed{50\ mW}$ of average Power

c) " " " absorbing 25 mVA " reactive "

d) Find complex Power for each element

$i_1 = \frac{50\angle 0\ mA\ (50)}{50+25+j50-j75} = \frac{25\angle 0^\circ}{75-25j} = \frac{2.5\angle 0^\circ}{79.01\angle -18.43^\circ} = 31.6\ mA\angle 18.43^\circ = 30 + j10$

$i_2 = i_S - i_1 = 50\angle 0 - 31.6\angle 18.43\ mA$
$= 20 - j10 = 22.36\angle -26.57^\circ\ mA$

$P_{50\Omega} = V_{RMS} I_{RMS} \cos(\theta_V - \theta_i) = (1.12)(22.36)\ mW = \boxed{25.0\ mW = P_{50\Omega}}$

$P_{25} = I^2 R = (31.6)^2 25 = \boxed{25\ mW = P_{25\Omega}}$

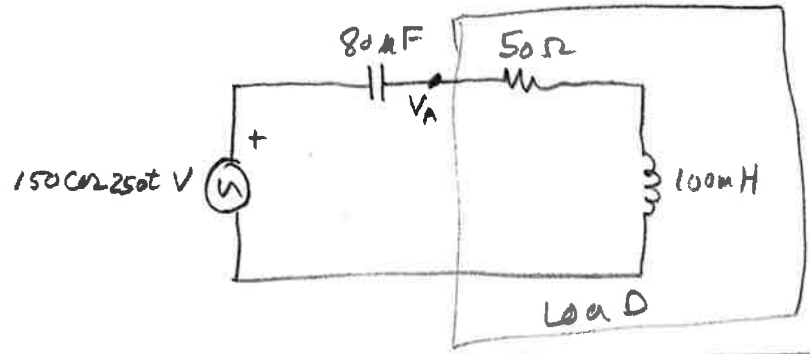
$P_{cap} = V_{RMS} I_{RMS} = I^2_{RMS} Z \cos(\theta_V - \theta_i)$
$= (31.6)^2(-75) = \boxed{-75\ mVAR = P_{cap}}$

$P_{IND} = I^2_{RMS} Z_{IND} = (31.6)^2 50 = \boxed{50\ mVar = P_{ind}}$

e) P_{AVG} delivered $= -50\ mW$; P_{AVG} absorbed $= 50\ mW$ ✓
f) P_{REAC} " $= 25\ mVAR$; P_{REAC} " $-75 + 50 = -25\ mVA$ ✓

10.20 Nilsson 11th

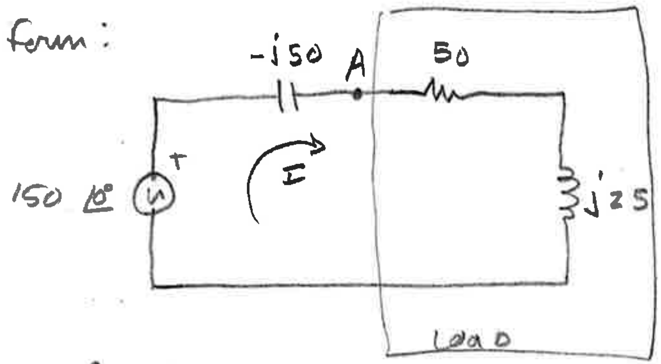
Find P_{Avg}
$P_{Reactive}$
P_{App}
absorbed by the load.



1) Put circuit in phasor form:

$Z_C = \frac{1}{j\omega C} = -j50$

$Z_L = j\omega L = 25j$



$I = \frac{150\angle 0}{-j50 + 50 + j25} = \frac{150\angle 0}{50 - 25j}$

$= \frac{150\angle 0}{55.90\angle -26.57} = 2.683\angle 26.57^\circ$ A
$= 2.4 + j1.20$ A

$V_A = I(50 + j25) = (2.683\angle 26.57^\circ)(55.90\angle 26.57^\circ)$
$= 149.98\angle 53.14$ V

then $P_{Avg} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \boxed{180\ W}$

$P_{Rea} = Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \boxed{90\ VARS}$

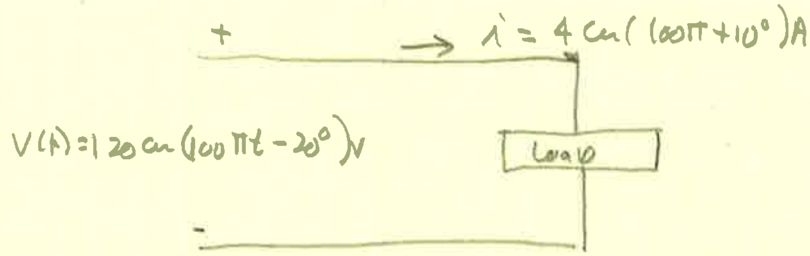
$P_{App} = P + jQ = 180 + j90 = \boxed{201.2\angle 26.57^\circ\ VA}$

also, $P_{Avg} = \frac{1}{2}I^2 R = \frac{1}{2}(2.683)^2(50) = 180W$ ✓

$P_{Reac} = Q = \frac{1}{2}I^2 Z_L = \frac{1}{2}(2.683)^2(25) = 90W$ ✓

Chap 10 Lecture

Apparant Power + Power Factor



Find the Apparent Power + PF of the load

$P_{App} = V_{eff} I_{eff} = \frac{1}{2} V_m I_m = \frac{1}{2}(120)(4) = \underline{\underline{240\,VA}}$

$P_{Ave} = \frac{1}{2} V I \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(4)(\cos(-20-10)) = 208\ W$

$PF = \frac{P_{Ave}}{P_{App}} = \underline{\underline{0.866}}$ (Leading, i.e. the current leads the voltage)

also, $\text{load} = \frac{V}{I} = \frac{120\angle -20}{4\angle 10} = 30\angle -30° = 26.0 - 15j = Z$

$PF = \cos(-30°) = \underline{\underline{.866}}$

Since current leads voltage, PF is <u>leading</u>

$-15 = -\frac{1}{\omega C} \Rightarrow C = \frac{1}{15\omega}$

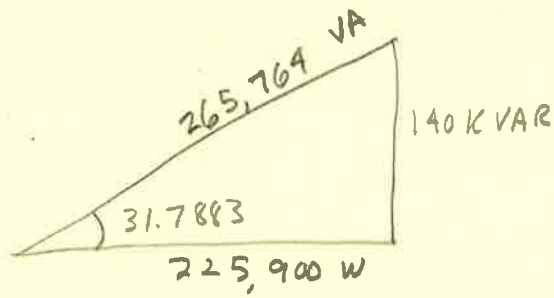
$C = \frac{1}{15(100\pi)} = 212\,\mu F$

$R = \underline{\underline{26.0\,\Omega}}$

$V = 110\,V_{RMS}$, 60 Hz

Load = 140 KVAR @ 0.85 PF lagging

what value of parallel capacitance is needed to correct to a unity PF



the capacitor must provide 140 KVAR of reactive power in order for unity PF

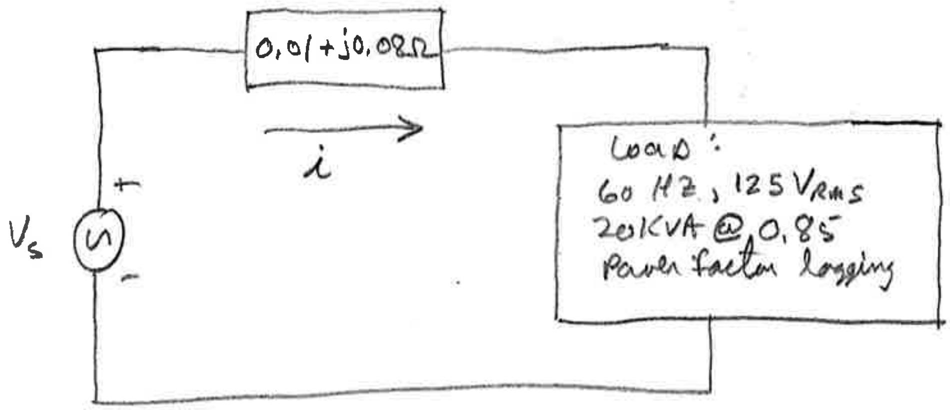
$$Q_C = \frac{V_{RMS}^2}{Z_C} = \frac{V_{RMS}^2}{1/\omega C} \quad \text{or} \quad C = \frac{Q_c}{\omega V^2} = \frac{140\,\text{KVAR}}{2\pi(60)(110^2)} = \underline{\underline{30{,}610\,\mu F}}$$

---

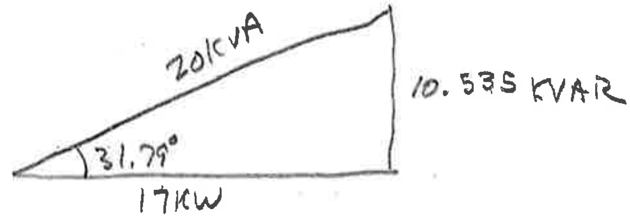
also, $$C = \frac{P(\tan\theta_{old} - \tan\theta_{new})}{\omega V_{RMS}^2}$$

$$= \frac{(225{,}900)(\tan 31.7883^\circ - \tan 0^\circ)}{2\pi(60)(110)^2}$$

$$= 30{,}691\,\mu F$$



a) Find V_{source}



$\theta = \cos^{-1} .85 = 31.79°$

$S = V_{RMS} I^{*}_{RMS}$

$17K + j\,10.535K = 125\angle 0 (I^{*})$

$I^{*} = 136 + j\,84.29 \ A_{RMS}$

$I = 136 - j\,84.29$

$V_s = 125\angle 0 + I(.01 + j.08) = \boxed{133.48 \angle 4.31° \ V_{RMS}}$

b) what is the average power loss in the feeder?

$P_L = |I^2| R = (160)^2 (.01) = \boxed{256 W}$

c) what size cap is need to make PF = 1?

method 1: $C = \dfrac{P(\tan\theta_1 - \tan\theta_2)}{\omega V^2_{RMS}} = \dfrac{(17K)(\tan 31.79 - \tan 0)}{2\pi(60)(125)^2} = \boxed{1788.7 \mu F}$

also, $\dfrac{V^2}{X_C} = Q \Rightarrow \dfrac{(125)^2}{X_C} = -10{,}535 \text{ VAR}$

But $X_C = -\dfrac{1}{\omega C} \Rightarrow C = -\dfrac{1}{|X_C| \omega} = -\dfrac{10{,}535}{-\omega(125)^2} \Rightarrow C = 1788.7 \mu F$

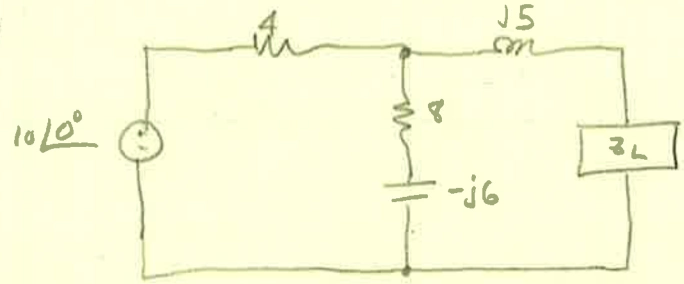
d) after the cap is installed, what is V_{source}?
after cap is installed, $Q = 0$ so $I = 136 \ A_{RMS}$

$V_s = 125 + 136(.01 + j.08) = \boxed{126.83 \angle 4.92° \ V_{RMS}}$

e) what is the new P_{avg} loss in the feeder?

$P_L = I^2 R = (136)^2 (.01) = \boxed{184.96 W}$

Determine Z_L to maximize power drawn



$$Z_{th} = j5 + 4 \,||\, 8-j6$$

$$= j5 + \frac{4(8-j6)}{4+8-j6} = j5 + \frac{32-24j}{12-j6} = j5 + \frac{40\angle -36.87^\circ}{13.416\angle -26.565^\circ}$$

$$= j5 + 2.9814\angle -10.3049^\circ$$

$$= j5 + (2.9333 - j.5333)$$

$$= 2.9333 + j\,4.4667\ \Omega$$

$$= 5.3437\angle 56.706^\circ$$

$$Z_L = Z_{Th}^* = \boxed{2.9333 - j\,4.4667\ \Omega}$$

$$V_{th} = \frac{10\angle 0\,(8-j6)}{4+8-j6} = \frac{10\angle 0\,(10\angle -36.87^\circ)}{13.4164\angle -26.5651} = \frac{100\angle -36.87}{13.4164\angle -26.5651}$$

$$V_{Th} = 7.4536\angle -10.3049^\circ\ V$$

$$P_{max} = \frac{V^2}{8R_{th}} = \frac{(7.4536)^2}{8(2.9333)}$$

$$\boxed{P_{max} = 2.3675\,W}$$